Content Specific Tips & Traps

1. **SIMULATION (#1).** You need to be able to describe how you will perform a simulation in addition to actually doing it.
* Create a correspondence between random numbers and outcomes. Assign Digits.
* Explain how you will obtain the random numbers (e.g., move across the rows of the random digits table, examining pairs of digits), and how you will know when to stop.
* Make sure you understand the purpose of the simulation -- counting the number of trials until you achieve "success" or counting the number of "successes" or some other criterion.
* Are you drawing numbers with or without replacement? Be sure to mention this in your description of the simulation and to perform the simulation accordingly.
1. **SIMULATION (#2).** If you're not sure how to approach a probability problem on the AP Exam, see if you can design a simulation to get an approximate answer.
2. **DIFFERENCE: Independent** eventsare not the same as **mutually exclusive** (disjoint) events**.**
	1. **INDEPENDENT:** Two events, A and B, are ***independent* if** the occurrence or non-occurrence of one of the events has no effect on the probability that the other event occurs. Formulaically, A & B are independent IFF P( A | B ) = P( A )
	2. **MUTUALLY EXCLUSIVE:** Events A and B are **mutually exclusive** if they cannot happen simultaneously**.** Formulaically, A & B are independent IFF P(A ∩ B) = 0.

**Example: Roll two fair six-sided dice.**

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| Let A = the sum of the numbers showing is 7, B = the second die shows a 6, and C = the sum of the numbers showing is 3. By making a table of the 36 possible outcomes of rolling two six-sided dice, you will find that P(A) = 1/6, P(B) = 1/6, and P(C) = 2/36. * Events A and B are **independent**. Suppose you are told that the sum of the numbers showing is 7. Then the only possible outcomes are {(1,6), (2,5), (3,4), (4,3), (5,2), and (6,1)}. The probability that event B occurs (second die shows a 6) is now 1/6. This new piece of information did not change the likelihood that event B would happen. Let's reverse the situation. Suppose you were told that the second die showed a 6. There are only six possible outcomes: {(1,6), (2,6), (3,6), (4,6), (5,6), and (6,6)}. The probability that the sum is 7 remains 1/6. Knowing that event B occurred did not affect the probability that event A occurs.
* Events A and B are **not disjoint**. Both can occur at the same time.
* Events B and C are **mutually exclusive** (disjoint). If the second die shows a 6, then the sum cannot be 3. Can you show that events B and C are not independent?
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**Symbolically:**

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| P ( A | B ) = P ( A ) and P( B | A ) = P( B ) IFF A & B are independent.Only the following formulas are provided on the AP Exam Formula Sheet:P(A∪B) = P(A) + P(B) – P(A∩B). AND P(A|B) = P(A∩B) / P(B) |

1. **DISCRETE RV’s:** Recognize a *discrete random variable* setting when it arises. Be prepared to calculate its mean (expected value) and standard deviation.

Example: Let X = the number of heads obtained when five fair coins are tossed.

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| --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | 5 |
| P(X) | 1/32=0.03125 | 5/32=0.15625 | 10/32=0.3125 | 10/32=0.3125 | 5/32=0.15625 | 1/32=0.03125 |

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1. **BINOMIAL.** Recognize **a *binomial situation* when it arises.**
The four requirements for a chance phenomenon to be a binomial situation are:

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| B | 1. On each trial, there are **two possible outcomes** that can be labeled "success" and "failure."
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| I | 1. The trials are **independent**.
 |
| N | 1. There are a fixed **number** of trials.
 |
| S | 1. The probability of a "**success**" on each trial is **constant**.
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Example:

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| Consider rolling a fair die 10 times. There are 10 trials. Rolling a 6 constitutes a "success," while rolling any other number represents a "failure." The probability of obtaining a 6 on any roll is 1/6, and the outcomes of successive trials are independent. Using the TI-83, the probability of getting exactly three sixes is (10C3)(1/6)3(5/6)7 or binompdf(10,1/6,3) = 0.155045, or about 15.5 percent. The probability of getting less than four sixes is binomcdf(10,1/6,3) = 0.93027, or about 93 percent. Hence, the probability of getting four or more sixes in 10 rolls of a single die is about 7 percent. If X is the number of sixes obtained when 10 dice are rolled, then If X is the number of 6's obtained when ten dice are rolled, then E(X) = x = 10(1/6) = 1.6667, and  |

Did you notice that the coin-tossing example above is also a binomial situation?

1. BINOMIAL vs. NORMAL: Realize that a binomial distribution can be approximated well by a normal distribution if the number of trials is sufficiently large. If n is the number of trials in a binomial setting, and if p represents the probability of "success" on each trial, then a good rule of thumb states that a normal distribution can be used to approximate the binomial distribution if np is at least 10 and n(1-p) is at least 10.
2. BINARY VS. GEOMETRIC. The primary difference between a binomial random variable and a geometric random variable is what you are counting**.** A binomial random variable counts the number of "successes" in n trials. A geometric random variable counts the number of trials up to and including the first "success."

Multiple Choice Questions

1. **A die is rolled 60 times with the following results recorded:**

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| --- | --- | --- | --- | --- | --- | --- |
| **Outcome** | **1** | **2** | **3** | **4** | **5** | **6** |
| **Frequency** | **10** | **6** | **12** | **9** | **8** | **15** |

The empirical probability of getting a 3 is

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 1/12
 | 1. 1/10
 | 1. 1/6
 | 1. 1/5
 | 1. 1/4
 |

1. **Two dice are rolled and the sum is recorded. Which of the following events has a probability equal to 1?**

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| --- | --- | --- | --- | --- |
| 1. S = {5 or 9}
 | 1. S ≥ 8
 | 1. S = 13
 | 1. S ≤ 13
 | 1. None of these
 |

1. **Which of the following could be considered binomial experiments?**
2. On each shot, a basketball players’ chance of scoring a free throw is estimated to be .38. A player tries 40 shots and the number of baskets is recorded.
3. On a specific island, it has been determined with probability .12 that an inhabitant carries a certain defective gene. Inhabitants are tested until 10 with the defective gene are found.
4. For a certain spinner, P(red)=1/3, P(green)=1/4, and P(blue)=5/12. The spinner is spun 100 times and the number of each color is recorded.

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| 1. I only
 | 1. II only
 | 1. I and III
 | 1. II only
 | 1. None of these
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1. **Suppose a box contains 3 defective light bulbs and 12 good bulbs. Two bulbs are chosen from the box without replacement. To find the probability that one of the bulbs drawn is good and one is defective, which expression would you use?**

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| --- | --- | --- | --- | --- |
| 1. $\frac{12}{15}+\frac{3}{14}$
 | 1. $\frac{12}{15}\left(\frac{3}{15}\right)+\frac{3}{15}\left(\frac{2}{15}\right)$
 | 1. $\frac{12}{15}\left(\frac{3}{14}\right)$
 | 1. $\frac{12}{15}\left(\frac{3}{14}\right)+\frac{3}{15}\left(\frac{12}{14}\right)$
 | 1. $\frac{12}{15}\left(\frac{11}{14}\right)+\frac{3}{13}\left(\frac{2}{12}\right)$
 |

1. **Which of the following is true about dependent events?**

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| 1. P(A|B) = 0
 | 1. P(A|B) = A
 | 1. P(A|B) = B
 | 1. P(A|B) = 1
 | 1. None of these
 |

1. **Identify why this assignment of probabilities cannot be legitimate: **
2. A and B are not given as mutually exclusive events
3. A and B are not given as independent events
4. is not known
5.  cannot be greater than either P(A) or P(B).
6. the assignment is legitimate.
7. **Which of the following events are mutually exclusive?**
8. A = the sum of two dice is 7.
9. B = the flip of a coin is a head.
10. C = the sum of two dice is 11.

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| --- | --- | --- | --- | --- |
| 1. A and B
 | 1. B and C
 | 1. A and C
 | 1. A, B and C
 | 1. No pairs are mutually exclus.
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**The probability of any person in your Student Council being selected for the Spring Dance Committee is 0.3, and the probability that a person on the Spring Dance Committee is elected Chairperson of this committee is 0.2.**

1. **Find the probability that a member of the Student Council, chosen at random, is the Chairperson of the committee.**

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| --- | --- | --- | --- | --- |
| 1. 0.2
 | 1. 0.3
 | 1. 0.5
 | 1. 0.6
 | 1. None of these
 |

1. **Find the probability that a member of the Student Council, chosen at random, is not the Chairperson of the committee.**

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| 1. 0.16
 | 1. 0.85
 | 1. 0.89
 | 1. 0.94
 | 1. None of these
 |

1. **Assume that the probability that a baseball player will get a hit in any one at-bat is .250. Which expression will yield the probability that his first hit will next occur at his 5th at bat?**

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| 1. 5C4 (0.25)4 (0.75)1
 | 1. (0.75)4 (0.25)1
 | 1. (0.25)4 (0.75)1
 | 1. 5C1 (0.25)1 (0.75)4
 | 1. None of these
 |

Free Response

1. Find the probability of flipping a head on a coin and rolling a sum of 8 on two dice.
2. A box contains 11 nickels, 4 dimes, and 5 quarters. If you draw 3 coins at random from the box without replacement, what is the probability that you will get a nickel, a dime and a nickel in that order.
3. A company estimates that 60% of the adults in the U.S. have seen its TV commercial and that if an adult sees the commercial, there is a 15% chance that the adult will buy its product. What is the probability that a randomly chosen adult has seen the company’s commercial and bought its product?
4. An analysis of the registered voters in the last primary indicated that 55% of the voters were women. Of the female voters, 35% are registered Democrats, 35% are registered Republicans, and the rest are assumed Independent. Of the male voters, the percentages are 30%, 45% and 25% (D,R,I). Find each probability
5. A voter chosen at random is a woman.
6. A voter chosen at random is a male Republican.
7. A Democrat chosen at random is male.
8. One hundred teenage boys and one hundred teenage girls were asked if they had ever made a purchase using the Internet. Thirty of the boys and sixty of the girls said they had made purchases. If one of these teenagers is selected at random,
9. what is the probability that he or she has made a purchase using the Internet?
10. what is the probability that the teenager us a girl, given that the person has made a purchase using the Internet.
11. Donald has ordered a computer and a desk from two different stores. Both items are to be delivered on Tuesday. The probability that the computer will be delivered before noon is .6 and the probability that the desk will be delivered before noon is .8. If the probability that either the computer or the desk will be delivered before noon is .9, what is the probability that both will be delivered before noon?

Multiple Choice KEY

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| 1 | **D** | 12 / 60 = 1 / 5.  |
| 2 | **D** | The entire population is represented by the sum of all possible combinations less than 13. |
| 3 | **A** | I is Binomial. II is geometric. III adhoc (all probabilities are different) |
| 4 | **D** | N = 12 good + 3 bad. If first is good, (12/15)(3/14) + 2nd is good (3/15)(12/14) = 0.1714 + 0.1714 = 0.3428Another way: $\frac{\left(\genfrac{}{}{0pt}{}{12}{1}\right)\left(\genfrac{}{}{0pt}{}{3}{1}\right)}{\left(\genfrac{}{}{0pt}{}{15}{2}\right)} = \frac{36}{105}=0.3428$ |
| 5 | **E** | If A & B are dependent, then none of these are ALWAYS correct.  |
| 6 | **D** | P(A∪B) is the intersection of P(A) and P(B). P(A∪B) cannot be greater than the sum of the two separate probabilities. |
| 7 | **C** | Σ two dice cannot be 7 and 11 at the same time.You can flip a coin independent of the roll of the dice, so they can happen at the same time |
| 8 | **E** | (0.3)(0.2) = 0.06.  |
| 9 | **D** | P(not chairperson) = 1 – P(Chariperson) = 1 – 0.06 = 0.94. |
| 10 | **B** | Geometric. P(fifth at bat = hit) = P(4 unsuccessful at bats) ⋅ P(1 successful)= (0.75)4 ⋅ (0.25)1 |

Free Response KEY

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| --- | --- |
| **1.** | P(head) and P(sum of 8) = $\frac{1}{2} ∙ \frac{5}{36}=\frac{5}{72}=0.0694$ |
| **2.** | P(nickel, dime, nickel in order) = $\frac{C\left(\genfrac{}{}{0pt}{}{11}{1}\right) C\left(\genfrac{}{}{0pt}{}{4}{1}\right) C\left(\genfrac{}{}{0pt}{}{10}{1}\right)}{P\left(\genfrac{}{}{0pt}{}{20}{3}\right)} = \frac{440}{6840}=0.0643$ |
| **3.** | P(commercial) = 0.60. P(buy product | commercial) = 0.15. P(commercial ∩ buy product) = P(commercial) x P (buy | commercial) = 0.60 x 0.15 = **0.09** |
| **4.** | (a) P(Women) = **0.55**(b) P(Male Republican) = 0.45 x 0.45 = **0.2025**(c) P(male|Democrate) = $\frac{\left(0.45\right)\left(0.30\right)}{\left(0.45\right)\left(0.30\right)+\left(0.55\right)\left(0.35\right)}=0.4122$ |
| **5.** | (a) P(purchased on Internet) = $\frac{\left(0.3\right)100+ \left(0.60\right)100}{200}=0.45$(b) P(girl|internet purchase) = $\frac{\left(0.60\right)100}{\left(0.3\right)100+\left(0.60\right)100}=\frac{2}{3}= 0.6666$ |
| **6.** | P(Computer before noon) = 0.6 P(desk before noon) = 0.80. P(computer OR desk before noon) = P(computer ∪ desk) 0.9P(C∪D) = P(C) + P(D) – P(C∩D). P(C∩D) = P(C) + P(D) - P(C∪D) = 0.6+0.8-0.9 = 0.5So probability of both delivered before noon = **0.5.** |