Content Specific Tips & Traps

1. **Correct Inference Procedure:** You must be able to decide which statistical inference procedure is appropriate in a given setting. Working lots of review problems will help you.
2. **KNOW YOUR Differences:** Know the difference between a population parameter, a sample statistic, and the sampling distribution of a statistic.

SPDC: State, Plan, Do, Conclude

HATS: Hypothesis, Assumptions, Test, Summarize

PHANCS: Problem, Hypothesis, Assumptions, Name (test), Calculate, Summarize (used at Northview H.S.)

1. **Tests of Significance :**

SPDC, HATS and PHANC . . .

★ Different mnemonics; same approach

|  |  |
| --- | --- |
| **H** | * Define the population of interest
* Identify the GREEK (μ, ρ or π, σ, α, … )
* Define the parameters in symbols or words.
* Identify KNOWN σ, UNKNOWN σ (using SX) or IMPLIED σ (using $\sqrt{\frac{p(1-p)}{n}}$ )
* Identify, or define a reasonable α-level
* State hypotheses in words and symbols.
* Choose correct inference procedure (type of test).
 |
| **A** | * Verify conditions for the correct inference procedure.
* The usual suspects: SRS, Independent (10% Rule), Normality/Large Sample
 |
| **T** | * Calculate the **test statistic** (show set-up for **t**, **z**, or **chi-square**) and the ***P*-value** (or rejection region).
 |
| **S** | * Draw a **conclusion** in context that is directly linked to your *P-value or rejection region*. Link your p-value to a reasonable alpha-level.
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1. **CONFIDENCE** INTERVALS:

|  |  |
| --- | --- |
| **H** | * Define the population of interest
* Identify the GREEK (μ, ρ or π, σ, … )
* Define the parameters in symbols or words.
* Identify KNOWN σ, UNKNOWN σ (using SX) or IMPLIED σ (using $\sqrt{\frac{p(1-p)}{n}}$ )
* Choose correct inference procedure (type of C-Level).
 |
| **A** | * Verify conditions for the correct inference procedure.
* The usual suspects: SRS, Independent (10% Rule), Normality/Large Sample
 |
| **T** | * Carry out the inference procedure (compute the interval).
 |
| **S** | * Interpret your results in the context of the problem.
* Use the Phrase-that-Pays®

“I am 95% confident that the interval produced captures the true population parameter. Or, 95 out of 100 random samples will produce an interval that captures the true population parameter. **IN CONTEXT!** |

1. **ASSUMPTIONS & CONDITIONS:** You need to know the specific conditions required for the validity of each statistical inference procedure -- confidence intervals and significance tests.
2. **ERRORS:** Be familiar with the concepts of Type I error, Type II error, and Power of a test.

Type 1 Error: Rejecting HO when it’s TRUE. P(Type I error) = α = significance level of the test.

Type II Error: Accepting the HO when it’s FALSE. P(Type II Error) = β

Power of the test: Probability of correctly rejecting HO.  Power = P(Type II Error) = 1 - β

Remember, in concept, there is an infinite number of HA

You can increase the power of a test by increasing the sample size (n) or increasing the significance level (α, the probability of a Type I error).

Multiple Choice Questions

1. **Which of the following are true?**

I. The mean of a population depends on the particular sample chosen.

II. The standard deviations of two different samples from the same population must be the same.

III. Statistical inference can be used to draw conclusions about populations based on sample data.

a. I and II b. II only c. III only d. II and III e. none of these

1. **A simple random sample of 100 high school seniors in a certain suburb reveals that 65% of them have at least part-time jobs in addition to school. If the expected value of this proportion is equal to the proportion of high school seniors who have at least a part-time job for the entire suburb, then we say the sample proportion is:**
2. a true value
3. an unbiased estimator of the population proportion
4. equal to the population proportion
5. and estimate whose variance equals the variance of data in the population
6. less than the population proportion since only 100 students were sampled.
7. **If the 90% confidence interval of the mean of a population is given by 45 +/- 3.24, which of the following is correct?**
8. There is a 90% probability that the true mean is in the interval.
9. There is a 90% probability that the sample mean is in the interval.
10. If 1,000 samples of the same size are taken from the population, then approximately 900 of them will contain the true mean.
11. There is a 90% probability that a data value, chosen at random, will fall in this interval.
12. None of these is correct.
13. **Two simple random samples of 50 undergraduates each from two universities are taken to determine the proportion of students who approve of the food service at their respective schools. The first university has an enrollment of 5,000 undergraduates while the second university has an enrollment of 35,000 undergraduates. Which of the following is the most accurate statement regarding these samples?**
14. The variability of the sample from the larger university will be greater than the variability of the sample from the smaller university.
15. The proportion of the students who approve of the food service will be the same since the sample sizes are the same.
16. The enrollment figures from the two universities are not relevant to whether the sample statistics are unbiased estimates of the parameters of the two populations.
17. If a university with 100,000 undergraduates conducted a simple random sample of 50 if its’ students, the results would be less accurate than either sample referenced above.
18. None of these is accurate.
19. **If a 95% confidence interval of the proportion of a population is 0.35 +/- 0.025, which of the following is NOT correct?**
20. If the sample size were to increase, the width of the interval would decrease.
21. An increase in confidence level generally results in an increase in the width of the confidence interval
22. This confidence interval could have been calculated after either a sample of a census was conducted.
23. If one would like a smaller confidence interval, one could increase sample size or decrease the confidence interval.
24. All of these are correct.
25. **The critical value for a 99.7% confidence interval for a population proportion is :**
26. 1 b. 1.96 c. 2 d. 2.78 e. 3
27. **From earlier studies, it is believed that the percentage of students favoring a four-day school week during May and June of every school year is approximately 85%. Which of the following sample sizes will create a margin of error no more than +/- 2% with 90% confidence?**
28. 79 b. 93 c. 146 d. 541 e. none of these
29. **During an AP Statistics class, the final answer in a “Statistical Jeopardy” game was “…to reduce the width of a confidence interval” Each of the student teams had 15 seconds to write the question for this answer. Their questions were:**

**TEAM A: Why should you increase sample size?**

**TEAM B: Why should you use a t-statistic instead of a z-statistic?**

**TEAM C: Why should you increase the confidence level?**

Which team(s) were correct?

1. A only
2. B and C
3. A and C
4. A,B, and C
5. All teams were wrong.
6. **Which of the following statements is correct?**
7. A sample statistic is significant if its population parameter is significant.
8. A sample statistic is significant if it is very unlikely that such a statistic could come from the population.
9. A sample statistic is significant if it can be established that it results from bias in the data-gathering process.
10. A sample statistic that is significant is always important.
11. None of these is correct.
12. **Which of the following is true?**
13. A highly significant result indicates that the sample result never really happened.
14. If the probability of sample data yielding a statistic as or more extreme than a given value if approximately 0, then we have a good indication that bias must have been involved with the data collection.
15. If the probability of sample data yielding a statistic as or more extreme than a given value is approximately 0, when we have a good indication that the value of the parameter could be significantly different from what is stated.
16. If the probability of sample data yielding a statistic as or more extreme than a given value is approximately 0, then we have a good indication of whoever stated the expected value was lying.
17. None of these is true.
18. **Given α = 0.05, which of the following is true?**
19. β = 0.95
20. The power of the test is 0.95.
21. α = P(rejecting Ho when Ho is true)
22. α = P(accepting Ho when Ho is false)
23. The value of β is independent of the value of α.
24. **Which of the following is NOT true?**
25. A way to reduce both Type I and Type II errors is to increase sample size.
26. If the sample size remains constant, then reducing α will increase the value of β.
27. α depends on the null hypothesis; β depends on both the null hypothesis and the value of the alternative hypothesis.
28. For the hypothesis test of a parameter, α must equal β.
29. All of these are correct.
30. **Which of the following is NOT a valid conclusion of the statement:**

**“The power of a hypothesis test for Ho: µ=10 when the truth is µ=12 is 0.91.”**

1. β = 0.09
2. The probability of accepting Ho when the true value of µ is 0.09.
3. 91% of the time this test will determine a false result with α = .05
4. 91% of the time this test will be sensitive to any alternative mean as different from 10 as 12 is.
5. All of these are valid

Free Response

1. In a recent survey of 1500 randomly selected U.S. adults, 68% if respondents agreed with statement, “I should exercise more than I do.”
2. Construct and interpret a 96% confidence interval to estimate the proportion of the U.S. adult population that would agree with this statement.
3. Explain what a 96% level of confidence means.
4. For this study, state one source of potential bias and how it would affect the estimate of the proportion of adults who would agree with the statement.
5. The administration of a very large high school have installed new drink vending machines in the cafeteria, in the main entrance to the school, and in the lobby area outside the gym. They would like to know if a student’s beverage choice depends upon location. A random sample of students over a one-week period was selected and interviewed. The results are shown in the table.

|  |  |  |  |
| --- | --- | --- | --- |
| Beverage | Cafeteria | Main Entrance | Gym |
| Soda | 22 | 8 | 41 |
| Juice | 19 | 5 | 11 |
| Water | 49 | 9 | 19 |

1. What should the administrators conclude? Support your answer with appropriate statistical evidence.
2. Is 71/183 a reasonable estimate of the proportion of all students in the high school who buy their beverages in the lobby of the gym? Explain your reasoning.

Multiple Choice KEY

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| 1 | **C** | A is **FALSE**. B is **FALSE**. C is the essence of what inference is.  |
| 2 | **B** | $\hat{p}$ (the statistic) is the **unbiased estimator** for p (the parameter) |
| 3 | **C** | A, B and D all refer to 90% **probability**. “C” is the Phrase-that-Pays®  |
| 4 | **C** | The N from the two populations does **not** affect the sample statistics. |
| 5 | **C** | A, B and D are all **TRUE**. C doesn’t make sense with the census. |
| 6 | **E** | InvNORM (0.0015. 0, 1) = -2.97. The Empirical Rule is ±**3σ** = 99.7% |
| 7 | **E** | ME = z\* $\sqrt{\frac{p(1-p)}{n}}$ where ME = 0.02, p = 0.85 and z\* = 1.645. Isolating n, **n = 863**  |
| 8 | **A** | A only. “B” is clearly **FALSE**. “C” acts to widen the C-Interval. |
| 9 | **B** | “B” is the **conceptual** **essence** of rejecting the HO |
| 10 | **C** | “A” contains the absolute word “never”. “B” refers to “bias”, which may or may not be present. “D” doesn’t make sense. “C” is the essence of **inference**. |
| 11 | **C** | By **definition**, α = P(rejecting Ho when Ho is true) |
| 12 | **D** | A, B and C are **TRUE**. D doesn’t make any sense. |
| 13 | **B** | β = P(Type II Error) where a Type II error is accepting HO when it is false.Since POWER = 1 - β, β = 1 – POWER. The POWER = 0.91, so β = 0.09 |

Free Response KEY

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| **1.** | Part (a)

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| Hypothesis / plan | Let p = true proportion of adults who agree with the statement. We want to find the 96% C-Interval for the estimated proportion of US adults that would agree with the statementn = 1500. $\hat{p}=68\%$ This is a 1-proportion z-test |
| Assumptions | The following conditions have been met for us to proceed with the 1-prop z-test:1. SRS. The situation specifically notes that the survey is a random selection. We will assume the survey was a simple random sample.
2. Large sample (10% rule). 10n = 10(1500) = 15000, which is less than the population of U.S. adults
3. Normality. np = (0.68)(1500) = 1020 AND n(1-p) = (0.32)(1500) = 480 are both well about the minimum requirement of 10.
 |
| Test | Critical value z\* = Invnorm(area = 0.02, μ = 0, σ = 1) = 2.054n = 1500. $\hat{p}=68\%$ Std.Error = $\sqrt{\frac{\hat{p} (1-\hat{p})}{n}}$ = $\sqrt{\frac{0.68 (.32)}{1500}}$ = 0.0120  | C-Internal = test statistic ± CV (std.dev of test stat)Since we don’t know the true proportion, we will use the std.error of the test rather than the std.dev)CI = 0.68 ± 2.054 (0.0120) = (0.6553, 0.7046) |
| Check: 1-PropZInt (1020, 1200, 0.96) = (0.65526, 0.70474) |
| Summary | We are 96% confident that the interval (0.655, 0.705) captures the true population parameter of estimated proportion of US adults that would agree with the statement about needing more exercise..  |

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Free Response Question 1 (continued)

Part (b):

The 96% level of confidence means that, if we where to take many samples of the same size from this population, about 96% of them will results in an interval that captures the actual parameter value.

Part (c): Source of potential bias:

1. Question design: “I should” is a leading prompt. It might lead to overestimation of “p”
2. Non-response bias could be a big factor. It is possible that those that did not respond share a similar characteristic that, if they were included could affect the results of the survey. Example: missing obese people could underestimate “p”

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| **2.** | Part (a)

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| Hypothesis / plan | Question to answer: Does students’ beverage choice depend on location?HO: beverage choice and location are independent.HA: beverage choice and location are not independent (are indeed associated)α is NOT defined, so we’ll assume α = 0.05 (1 in 20)This s a χ2 test of independence. |
| Assumptions | The following conditions have been met for us to proceed with χ2 test of independence:1. SRS. The situation specifically notes that the survey is a random selection. We will assume the survey was a simple random sample.
2. Large sample (10% rule). 10n = 10(183) = 1830, which is assumed to be less than the total number of servings sold from the vending machines
3. Expected counts are [B] = $\begin{matrix}34.9&8.58.&27.55\\17.2&4.2&13.6\\37.9&9.3&29.9\end{matrix}$. Not all the expected values are ≥ 5, so we’ll proceed with caution.
 |
| Test | n = 183d.freedom = 4 | χ2 = $\sum\_{}^{}\frac{(Observations-expected)^{2}}{Expected}$ = $\frac{(22-34.9)^{2}}{34.9}$ + …= 19.445P(χ2 > 19.445) = 0.000642 |
| Summary | Because of P-Value of 0.000642 is less than our α = 0.05 significance level, we reject HO. We have convincing evidence that there is association between the location of the vending machine and the types of beverages purchased,  |

 |

Part (b). It would **NOT** be reasonable to use the ratio 71/183 as an estimate of all students in the high school who buy their beverages from the lobby of the gym. The week may not represent the normal operations of the school or of the gym. For instance, if the chosen week of sampling were during multiple sporting events held in the gym, the numbers would be inflated, and would include ALL consumption (students, spectators, coaches, etc.). The opposite could also be true; if there were no sporting events, the overall consumption could be understated.